Intro to Traffic (Flow) Phenomena and (Macroscopic) Modeling

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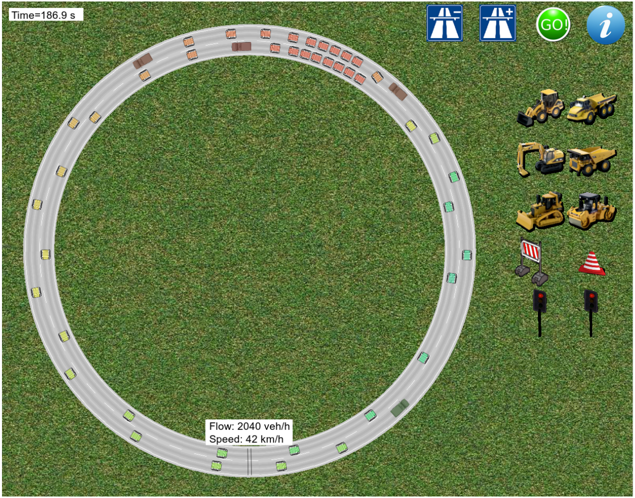
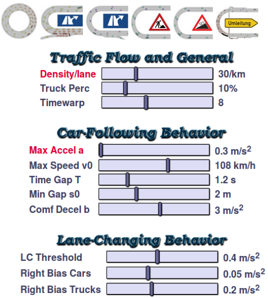
November 2019

Overview

1. Traffic Flow Dynamics: Instability and Traffic Breakdown
2. Collective Dynamics: Empirical Data and Congestion Patterns
3. Traffic Modeling: Dynamics vs. Planning
4. Traffic Observables and Flow-Density Relation
5. Macroscopic Modeling: Fundamental Diagram and First-Order Models

Traffic Flow Breakdown: Microscopic Simulation





- Change the road geometry by dragging
- Click onto the road to disturb traffic flow
- Drag obstacles or construction vehicles to create new bottlenecks
- Drag traffic lights to the road and click on them to toggle between red and light
- Use the info button repeatedly for more info

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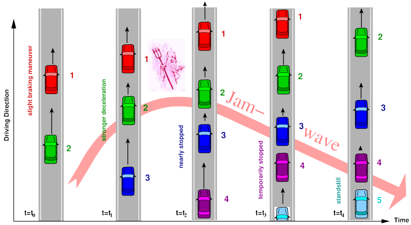
Instability: Perturbations grow to Stop Waves



▸ www.traffic-simulation.de

▸ Sugiyama et al. (2008)

- *Delayed* reaction to slight braking maneuver requires *stronger* reaction
- Growing perturbations in *upstream* direction
- Wave propagates against driving direction with about -15 km/h:

$$c_{\text{prop}} = \frac{\text{vehicle length + minimal gap}}{\text{time gap}} \approx \frac{6\,\mathrm{m} + 3\,\mathrm{m}}{1.8\,\mathrm{s}} = 5\,\frac{\mathrm{m}}{\mathrm{s}} = 18\,\frac{\mathrm{km}}{\mathrm{h}}$$

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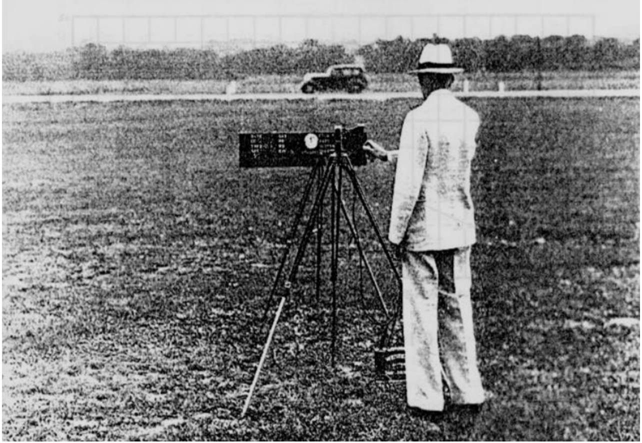
First Measurements (Greenshields, 1933/35)



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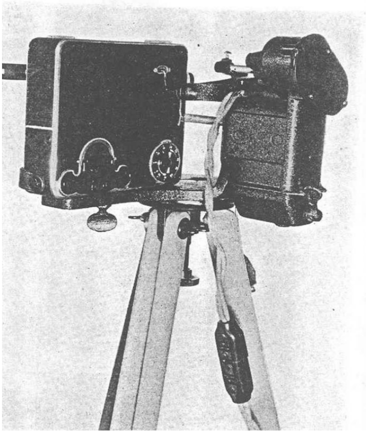

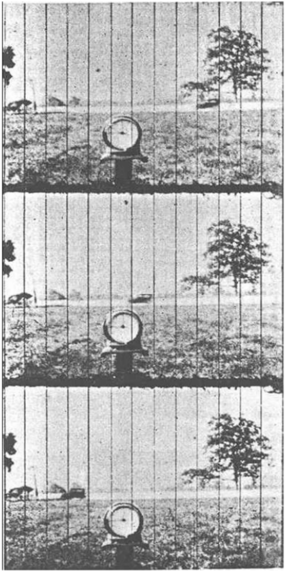

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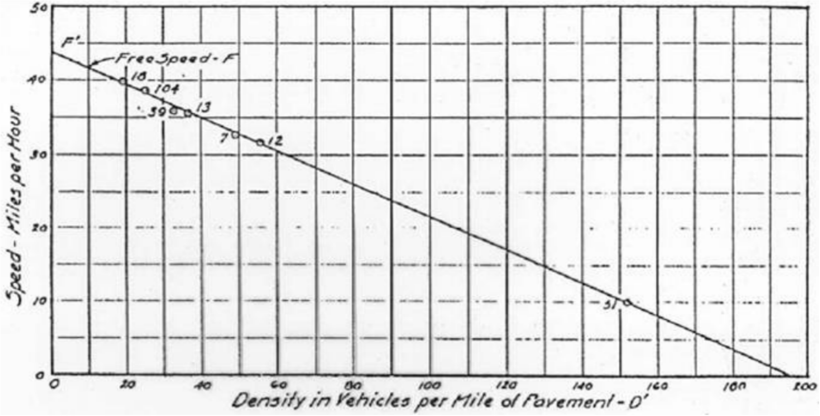

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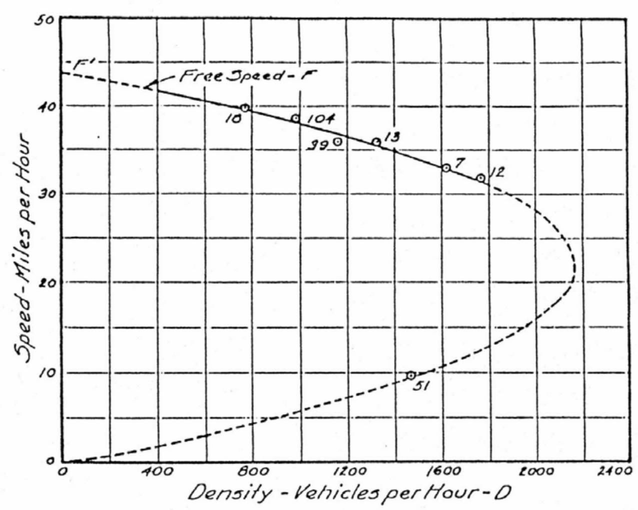
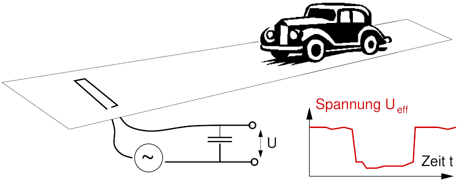
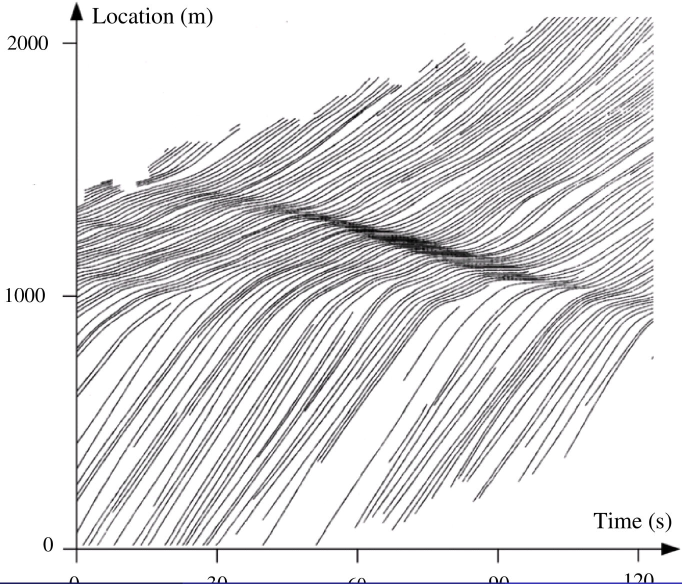


Cross-Sectional Measurements by Induction (e.g. A5/Frankfurt)

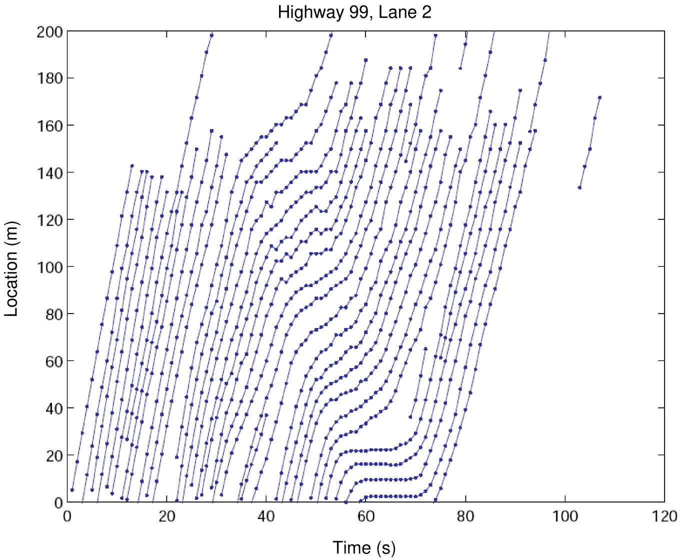


http://traffic-flow-dynamics.org/traffic-states

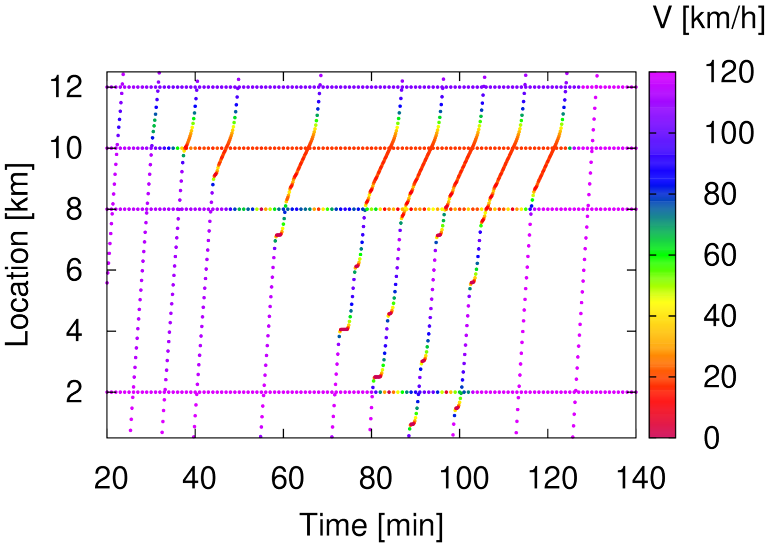
Trajectory Data: Treiterer (1970)



Trajectory Data: Coifman (Highway 99, second lane)



Sample of Trajectories: Floating-Car Data (FCD)



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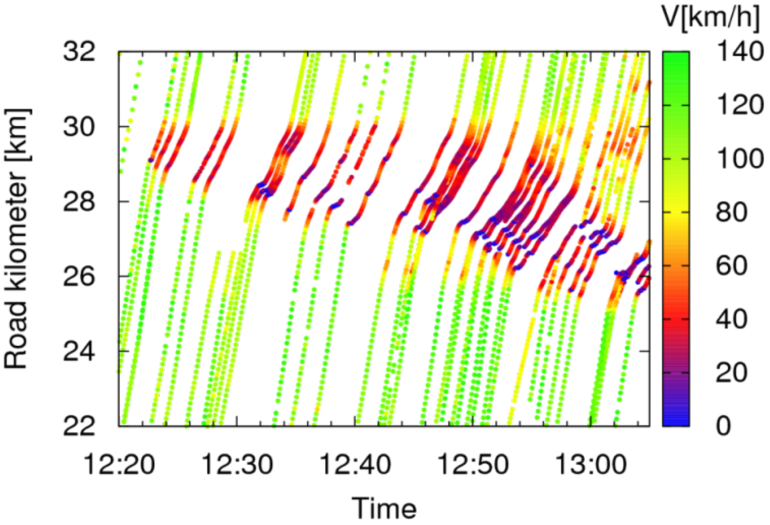
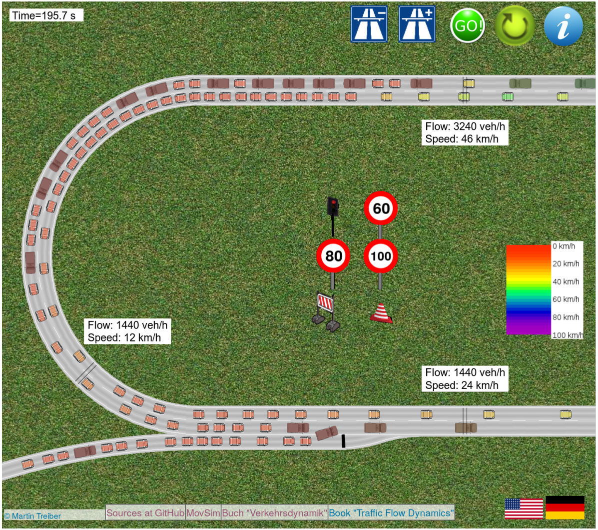
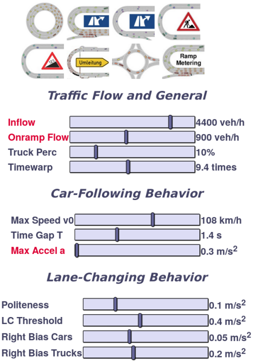


Open System: Traffic Flow Dynamics at Bottleneck





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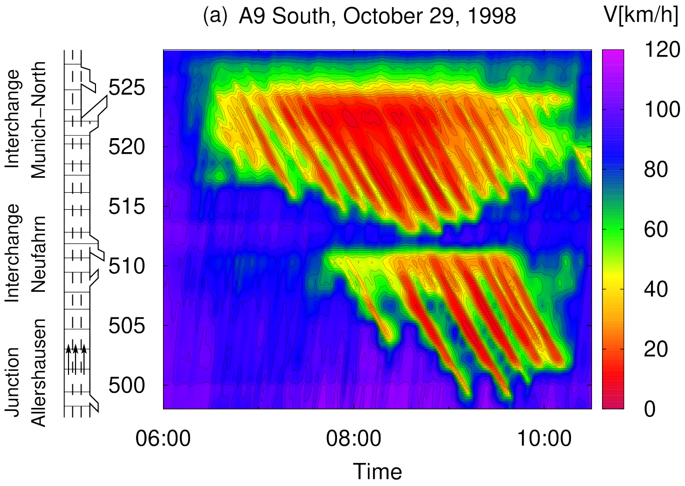
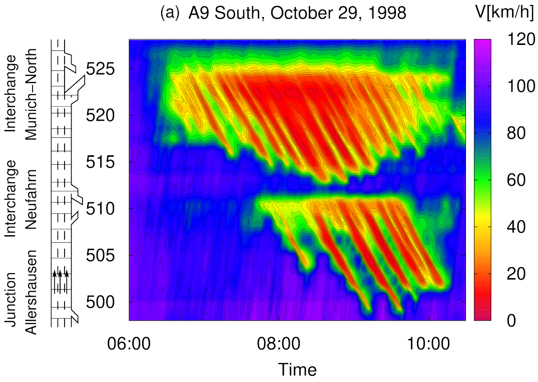
Spatio-Temporal Dynamics from Loop Detector Data

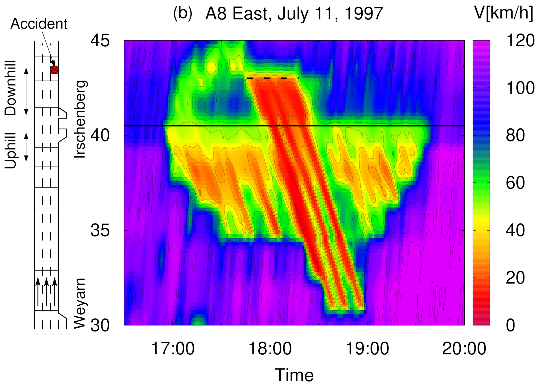


Characteristic Speeds of Traffic Jam Fronts



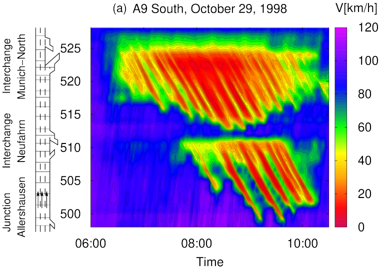
- Downstream jam front:
  - Fixed (at bottleneck): $V = 0$
  - Moving: $V \approx -15\,\mathrm{km/h}$
- Upstream jam front
  - No charact. speed
  - Balance of in-/out-flow
  - (demand / supply)
- Frequency of waves:
  - Depends on bottleneck strength

Flow Breakdown at Bottlenecks



- Flow Breakdown at bottlenecks
- Permanent at fixed locations:
  - on-/off-ramps, uphill gradients, traffic light, etc.
- Spontaneous:
  - accident, etc snowplough, etc.
  - (Moving bottleneck: snowplough)

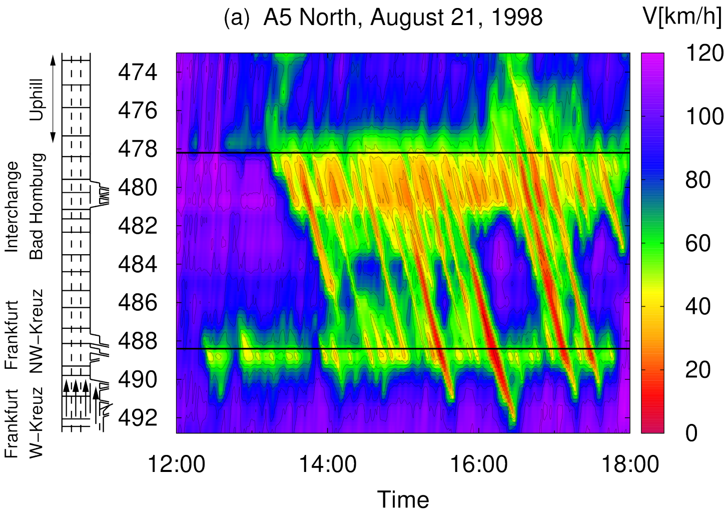
Definition of Bottleneck



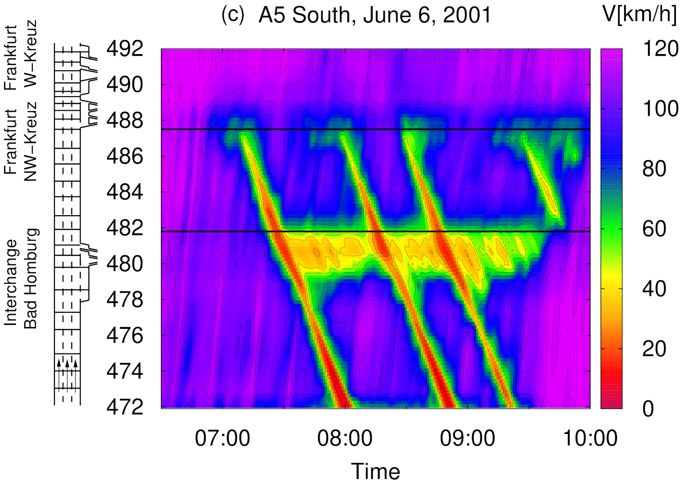
Bottleneck

We define a *bottleneck* as a local reduction of the road capacity. Bottlenecks can be *permanent* attributes of the infrastructure (e.g., on-ramps, off-ramps, roadworks, etc.) or *temporary*, e.g., when caused by accidents.

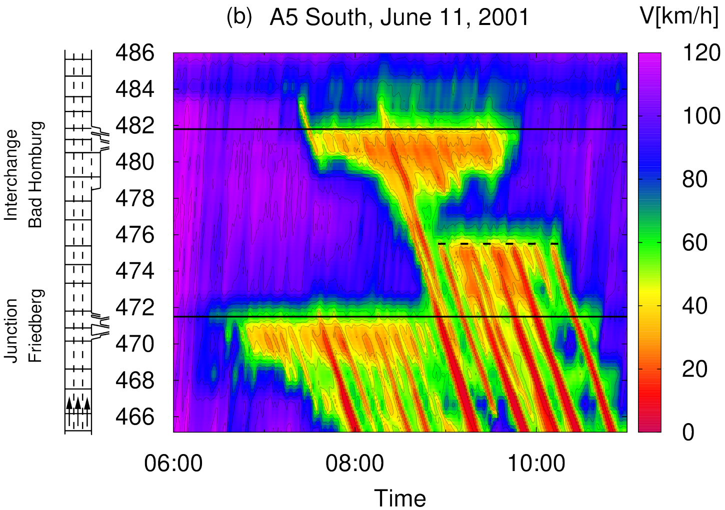
Characteristic speed for *all* perturbations: $-15 \pm 3\,\mathrm{km/h}$



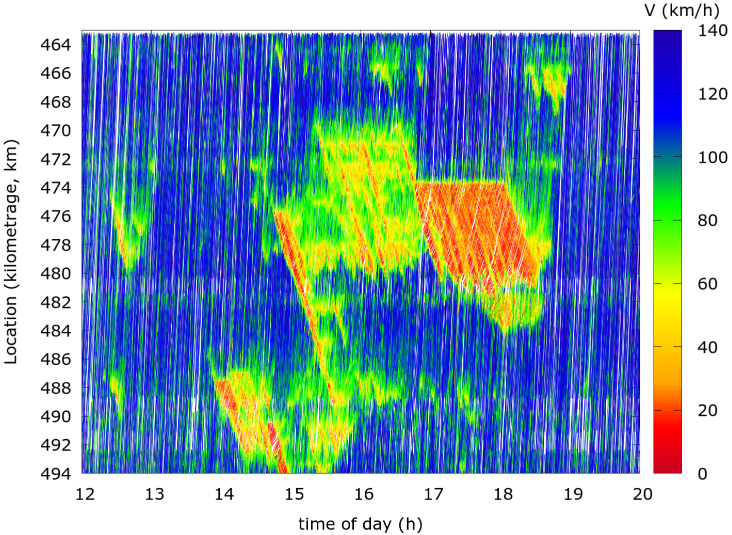
Localized Congestion Patterns: Pinned or Moving



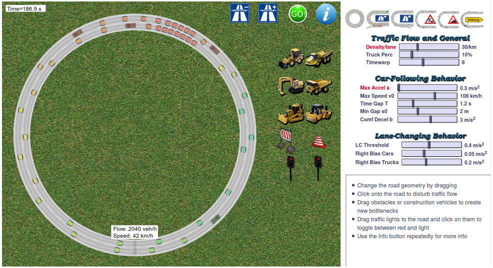
Rich Spatio-Temporal Congestion Patterns (Loop Data)



Rich Spatio-Temporal Congestion Patterns (FCD)



Three Ingredients for Traffic Flow Breakdown



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3 Conditions for Traffic Flow Breakdown

- High traffic load (pre-condition for propagating perturbations)
- Active Bottleneck ('weakest link')
- Disturbances caused by individual drivers (as trigger)

Overview

Traffic Flow Models

Time Scale	Field	Models	Aspect of Traffic (examples)
$< 0.1\,\mathrm{s}$	vehicle dynamics	sub-microscopic	control of engine and brakes
$1\,\mathrm{s}$ $10\,\mathrm{s}$ $1\,\mathrm{min}$ $10\,\mathrm{min}$	traffic flow dynamics	car-following models macroscopic models	reaction time, time gap acceleration and deceleration cycle period of traffic lights stop-and-go waves
$1\,\mathrm{h}$ $1\,\mathrm{day}$ $1\,\mathrm{year}$ $5\,\mathrm{years}$ $50\,\mathrm{years}$	transportation planning	route assignment traffic demand statistics age pyramid	peak hour daily demand pattern building/changing infrastructure socioeconomic structure demographic change

Traffic Modeling Aspects: Dynamics $\leftrightarrow$ Planning

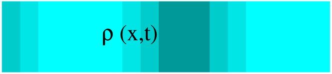
- Common: Time-dependent traffic phenomena
- *Temporal:* Minutes/hours $\leftrightarrow$ hours/days/years
- *Objective:* Externally given demand and infrastructure $\leftrightarrow$ Dynamics of *demand*, changes in infrastructure, policies, land-use
- *Subjective:* Human (automated) driving behavior (accelerating, braking, lane-changing, turning, etc) $\leftrightarrow$ Trip, mode, route, ... choice

Traffic *Flow* Dynamics

Collective effects and phenomena arising from accelerating, braking, lane-changing behavior of vehicle-driver units

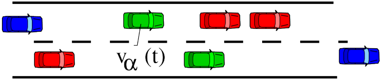
Model Categories

Macroscopic Model



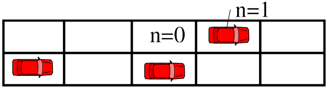
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_e(\rho)) = 0$$

Microscopic Model



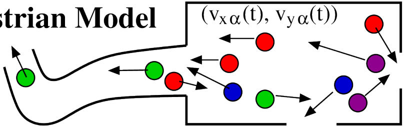
$$\frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = a_\alpha(s_\alpha, v_\alpha, \Delta v_\alpha)$$

Cellular Automaton (CA)



$$n_j(t+1) = F(\{n_k(t)\})$$

Pedestrian Model

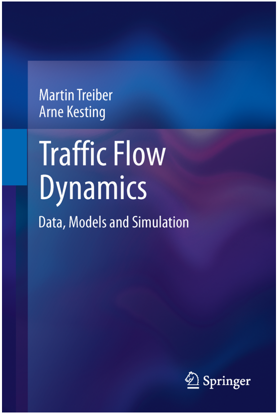


$$\frac{\mathrm{d}\vec{v}_\alpha}{\mathrm{d}t} = \vec{a}_\alpha\left(\vec{v}_\alpha, \vec{v}_{0\alpha}, \{\vec{x}_\beta\}, \text{Walls} \ldots\ \right)$$

Mathematical Structure

- **Partial differential equations**
  Both location x and time t continuous as independent vars, e.g. $V(x,t)$ (Lighthill-Witham-Richards, 1955/56)
- **Coupled ordinary differential equations**
  Continuous state vars depend on time only, e.g. $v_\alpha(t)$, and on other vehicles. *Car-following models*, e.g. Intelligent Driver Model, 2000
- **Coupled iterated maps**
  Discrete time steps Δt (as *parameter*) with continous state, e.g. $x_\alpha(t)$, $v_\alpha(t)$
- **Celluar Automata**
  All variables discrete: Space devided into fixed cells, time update in fixed intervals, e.g. Nagel-Schreckenberg model, 1992
- **Discrete state, continous in time**
  e.g. lane-changing models (with integer-based lane index)
- **Static models (*traffic stream models*)**
  Pair-wise relations between macrosopic state vars, e.g. fundamental

References



▸ http://traffic-flow-dynamics.org

▸ www.verkehrsdynamik.de

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Traffic Flow Observables

Definition (Traffic Flow Q)

Number of vehicles ΔN passing a cross-section at x within a time interval Δt:

$$Q(x,t) = \frac{\Delta N}{\Delta t}$$

Definition (Average Speed V)

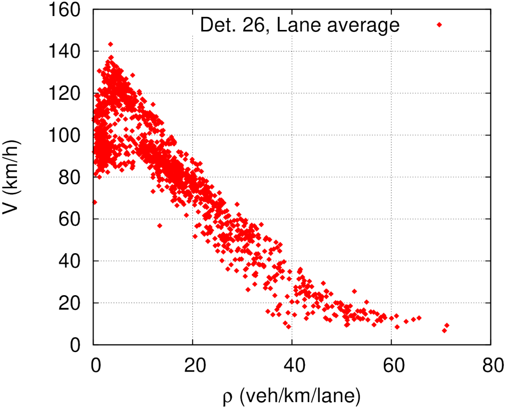
(Arithmetic) mean speed of the ΔN vehicles passing a cross-section at x during an aggregation interval Δt.

Definition (Traffic Density ρ)

Number of vehicles N on a road segment Δx at a given time t:

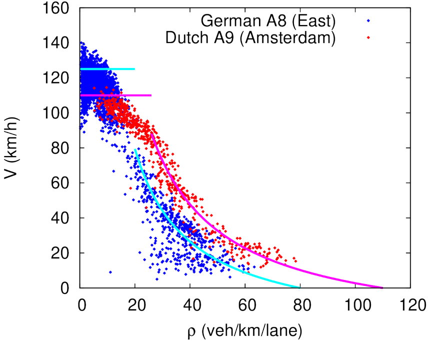
$$\rho(x,t) = \frac{N}{\Delta x}$$

Speed-Density Relation from Loop Detectors (all lanes)



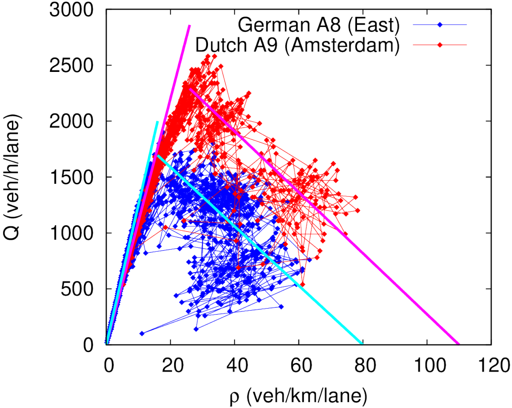
Why does the average speed decrease again for small densities?

Speed-Density Relation: Desired Speed



- $\rho \approx 0$: drivers usually not influenced by others
- For $\rho \to 0$: average *free speed* V_0 (also: *desired speed*)
- Minimum of actual desired speed, physically possible attainable speed, speed limit

Flow-Density Relation



Determine the maximum density ρ_{max}, the maximum flow, and the *capacity drop* using the fit lines.

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Macroscopic (First Order) Models

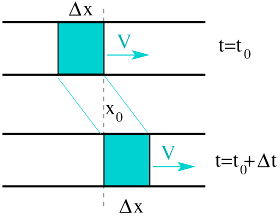
- Models for spatio-temporal dynamics of $\rho(x,t), Q, V$
- Foundations of every macroscopic traffic model:
  - Hydrodynamic Relation $Q = \rho V$
  - Continuity equation: derived from conservation of vehicle flows
  - Both equations parameter-free $\rightarrow$ hold for *any* macroscopic model

1. Lighthill-Witham Richards (LWR) Models
2. LWR for triangular fundamental diagram

Hydrodynamic Flow-Density Relation

- 'Flow equals density times speed'

$$Q(x,t) = \rho(x,t)V(x,t) \qquad \text{check: } \frac{N}{\Delta t} = \frac{N}{\Delta x}\frac{\Delta x}{\Delta t}$$



Derivation: $\Delta n = \rho \Delta x$ vehicles pass x_0 in $\Delta t = \Delta x / V$

$$\Rightarrow \text{for } x_0: \quad Q = \frac{\Delta n}{\Delta t} = \frac{\rho \Delta x}{\Delta t} = \rho V$$

Lighthill-Whitham-Richards Model (1955/1956)

- Continuity equation is partial differential equation for ρ and V:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = 0$$

- Describes rate of change in density in terms of gradients of flow
- With hydrodynamic relation $Q = \rho V$ third quantity can be derived
- Additional equation for flow (or speed) needed to 'complete' the model

Macroscopische First Order Models

Since the continuity equation is completely determined by the geometry of the road infrastructure, the macroscopic models differ in their modeling of speed or flow, only.

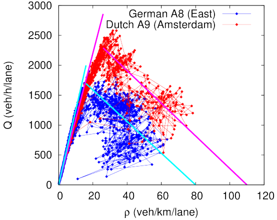
Lighthill-Whitham-Richards Model (1955/1956)

- Assumption: Q or V always in local equilibrium w.r.t. actual density
- Q resp. V follows *instantaneously* ρ:

$$Q(x,t) = Q_\mathsf{e}(\rho(x,t)) \quad \text{resp.} \quad V(x,t) = V_\mathsf{e}(\rho(x,t)).$$

- $Q(x,t)$ and $V(x,t)$ coupled to $\rho(x,t)$ (*static equilibrium*)
- Speed-Density relation: $V_e(\rho)$
- Fundamental diagram: $Q_e(\rho) = \rho V_e(\rho)$

Flow-Density Relation and Fundamental Diagram



Definition

The *flow-density diagram* represents aggregated empirical data that generally describes *non-stationary heterogeneous* traffic, i.e., different driver-vehicle units far from equilibrium.
The *fundamental diagram* describes the theoretical relation between density and flow in *stationary homogeneous* traffic, i.e., the steady state equilibrium of identical driver-vehicle units.

Lighthill-Whitham-Richards Model (LWR)

Continuity equation (homogenous road segment)

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0$$

With chain rule

$$\frac{\partial Q_e}{\partial x} = \frac{\mathrm{d}Q_e(\rho)}{\mathrm{d}\rho} \frac{\partial \rho}{\partial x}$$

$\Longrightarrow$

LWR Model (homogenous road segment)

$$\frac{\partial \rho}{\partial t} + \frac{\mathrm{d}Q_e(\rho)}{\mathrm{d}\rho} \frac{\partial \rho}{\partial x} = 0$$

- Inhomogenous segment or on-/off-ramps $\rightarrow$ additional terms
- $Q_e(\rho)$ not specific $\rightarrow$ LWR Model *class*
- Only *one* dynamic equation $\rightarrow$ *First-order* models

Propagation of Density Variations

- *Non-linear wave equations* describe so-called *kinematic waves*
- Propagation velocity $\tilde{c}$ for those waves (density variations)

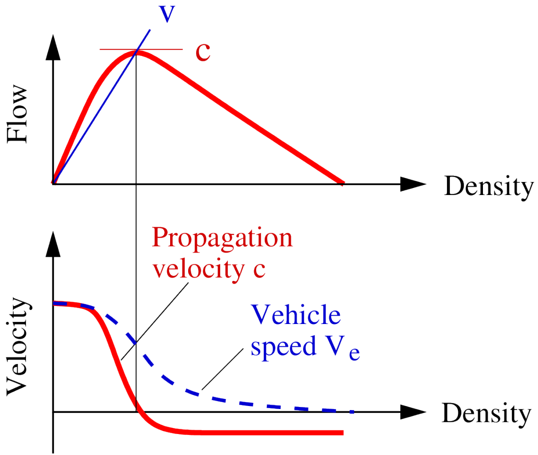
$\Longrightarrow$

$$\tilde{c}(\rho) = \frac{\mathrm{d}Q_e}{\mathrm{d}\rho} = \frac{\mathrm{d}(\rho V_e(\rho))}{\mathrm{d}\rho}$$

- $\tilde{c}$ depends on density
- Proportional to gradient of fundamental diagram

$\rightarrow$ Density variations propagate *in* or *against* driving direction

Propagation of Density Variations



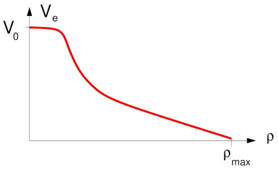
Relative Propagation Velocity w.r.t. Vehicle Speed

With

$$\tilde{c}(\rho) = Q_e'(\rho) = \frac{\mathrm{d}(\rho V_e(\rho))}{\mathrm{d}\rho} = V_e(\rho) + \rho V_e'(\rho)$$

$\Rightarrow$ relative velocity w.r.t. vehicle speed V

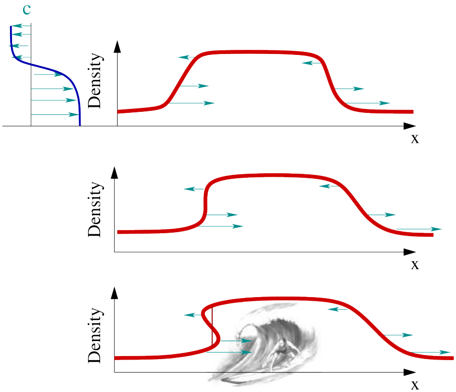
$$\tilde{c}_{\mathrm{rel}}(\rho) = \tilde{c}(\rho) - V = \tilde{c}(\rho) - V_e(\rho) = \rho V_e'(\rho)$$



- $V_e'(\rho) \leq 0 \Rightarrow$ density perturbations propagate *backwards* from drivers' perspective
- Microscopic view: drivers react only to leading vehicle but not to vehicles behind

Shock Waves in LWR

LWR equation describes density waves with different propagation velocities (the lower the density, the higher the propagation velocity)

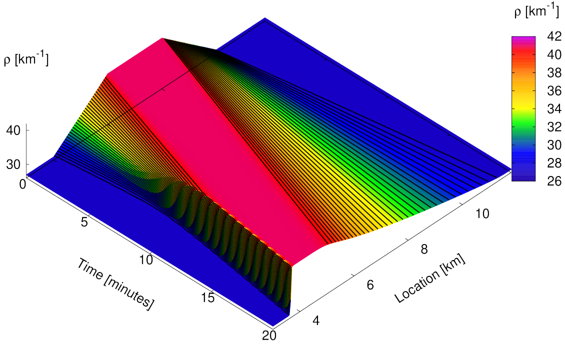


⇒ Upstream wave front becomes steeper while downstream front disperses

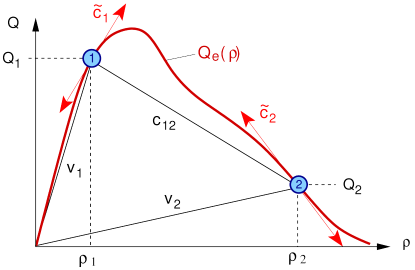
Shock Waves in LWR

- Transition from free → congested: traffic becomes discontinuous
- Vehicles from congested → free: 'traffic' accelerates less and less (*dispersion fan*)

⟹ LWR unrealistic

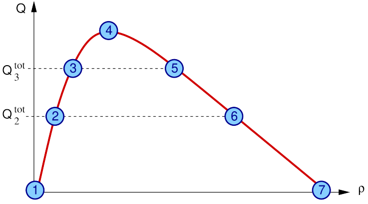


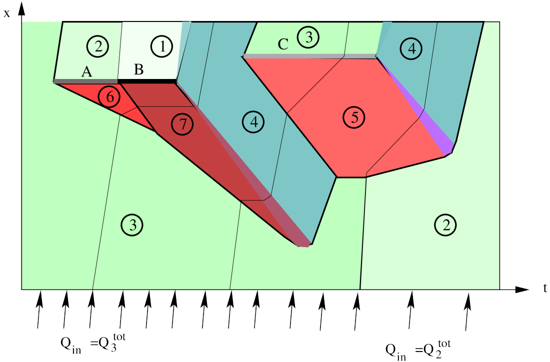
Three Characteristic Velocities in LWR



1. The **propagation velocity of density variations** $\tilde{c}(\rho) = Q_e'(\rho)$ is given by the slope of the fundamental diagram.
2. The **propagation velocity of shock fronts** c_{12} is given by the slope of the secant connecting points of the fundamental diagram corresponding to traffic on either side of the front.
3. The **vehicle speed** $V_e = Q_e(\rho)/\rho$ is given by the slope of the secant connecting the origin with the corresponding point on the fundamental diagram.

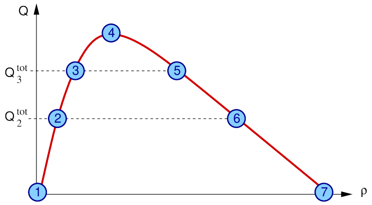
Example: Spatio-Temporal Dynamics in LWR

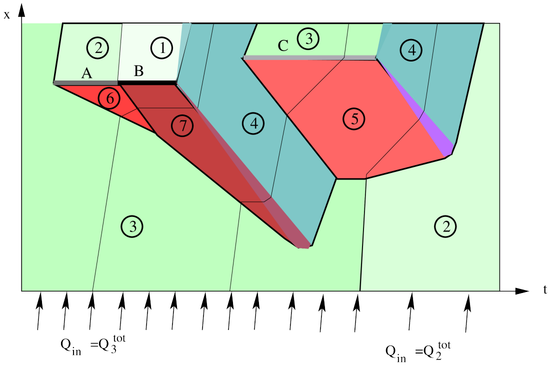




- Inflow: Q_{in} at ③ (later decrease to ②)
- Temporary bottlenecks: A with capacity $K_A = Q_2^{\text{tot}}$, B full road block with $K_B = 0$, and C with capacity $K_C = Q_3^{\text{tot}}$
- 3 trajectories indicating vehicle speeds
- Transition from higher to lower densities: 'soften' over time while remaining discontinous (shocks)

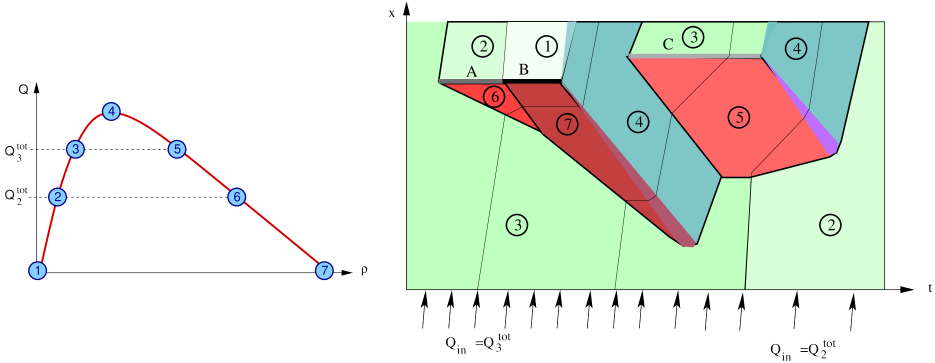
Example: Spatio-Temporal Dynamics in LWR





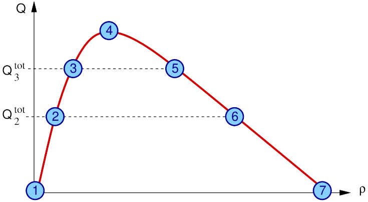
- A reduces flow to $K_A = Q_2^{\text{tot}}$, since $Q_{\text{in}} > K_A \rightarrow$ traffic congestion
- Downstream of bottleneck free traffic (Zustand ②)
- Congestion upstream (Zustand ⑥), jam front with $c_{36} < 0$, stationary at bottleneck $c_{62} = 0$
- Velocity in congested traffic $V_6 = Q_6/\rho_6 > 0$

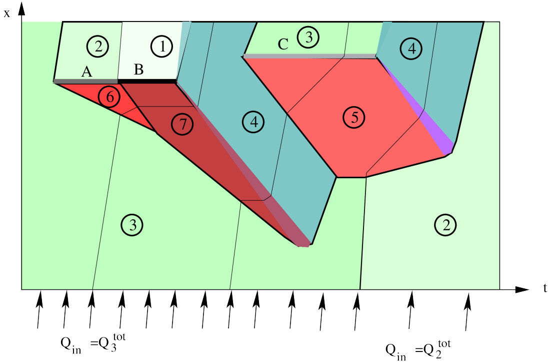
Example: Spatio-Temporal Dynamics in LWR



- Full road block B reduced flow up- and downstream to 0
- Upstream: maximum density (Zustand ⑦), downstream empty road (Zustand ①)
- Transition speeds $c_{67} < c_{37} < c_{36} < 0$ and $c_{71} = 0$ of course

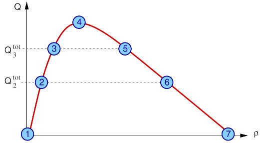
Example: Spatio-Temporal Dynamics in LWR

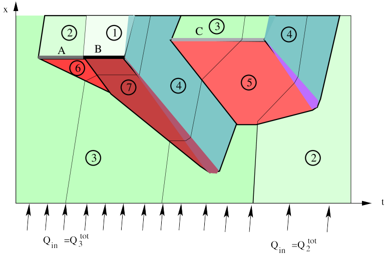




- Blockage B deactivated with transition velocity $c_{74} \approx c_{67} < 0$
- Transition from standstill to maximum (out-)flow
- (Softening (*dispersion*) often unrealistic)

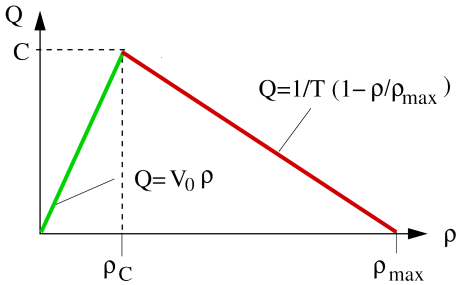
Example: Spatio-Temporal Dynamics in LWR





- C: up- and downstream flows $Q_3^{\text{tot}} = Q_5^{\text{tot}}$, $\rightarrow c_{53} = 0$
- Growing traffic jam propagates in upstream direction with $c_{45} < 0$
- Stationary transition $c_{35} = 0$ at bottleneck
- Reduced inflow: shrinking traffic jam with downstream front velocity $c_{25} > 0$
- Deactivation of bottleneck: transition from congested $\rightarrow$ maximum flow progages backwards with $c_{54} < 0$
- Until reaches upstream jam front $\rightarrow$ complete resolution of traffic jam

LWR with Triangular Fundament Diagram



$$Q_{\mathsf{e}}(\rho) = \begin{cases} V_0\rho & \text{falls } \rho \le \rho_{\mathsf{K}} \quad \text{(free traffic)} \\ \frac{1}{T}\left[1 - \frac{\rho}{\rho_{\max}}\right] & \text{falls } \rho_{\mathsf{K}} < \rho \le \rho_{\max} \quad \text{(congested traffic)} \end{cases}$$

- Easy to solve
- Two propagation velocities for perturbations, and no dispersion
- *Section-Based Model*, discrete version: *Cell-Transmission Model (CTM)*

Beyond LWR: Macroscopic Models with Dynamic Velocity

- LWR with *single* dynamic equation, $V(x,t)$ without *own* dynamics
- Instantaneous adaptation of vehicle speed unrealistic (finite acceleration)
- LWR without *traffic instabilities* (growing stop-and-go waves)
- $\rightarrow$ Not velocity but acceleration as function of traffic situation
- V second dynamic variable (*second-order models*)